Labor Market Dynamics When Ideas are Harder to Find *

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Abstract

This paper evaluates the impact of slowing economic growth on labor market dynamism and misallocation. It provides a model of endogenous growth via imitation in a frictional labor market. The framework accounts for rich data on worker job-to-job transitions as well as stochastic and lifecycle properties of firm growth and job reallocation. High productivity entrants gradually replace obsolescing incumbents by poaching their workers, a process that is intermediated via a frictional labor market. When the likelihood of entrants imitating technologies in the tail of the distribution falls (ideas are harder to find), so does growth. Consistent with US data over the past 30 years, firm entry, incumbents' employment response to productivity shocks, and job-to-job transitions decline, while the share of old firms increases. With lower imitation, however, there is *less* misallocation, because the slower aggregate rate of obsolescence induces productive firms to invest more in costly hiring and grow faster to their optimal size.

Keywords: Business Dynamism, Endogenous Growth, Firm Dynamics, Imitation, Job Turnover, Marginal Surplus, Misallocation, Net Poaching, On the Job Search, Search Frictions, Unemployment, Vacancies, Worker Flows.

JEL Classification: D22, E23, E24, E32, J23, J63, J64, J69, O40, O47.

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1 Introduction

Economic growth has slowed down in the U.S. One prominent explanation is that *ideas are getting harder to find* (Bloom, Jones, van Reenen, and Webb, 2020) and, as a result, aggregate productivity growth is now weaker (Gordon, 2016; Fernald, 2016). At the same time, business and labor market dynamism have declined along a host of margins. Firm entry has fallen, firm employment has become less responsive to productivity shocks, job reallocation has decreased (Decker, Haltiwanger, Jarmin, and Miranda, 2020), and job-to-job transitions have declined (Fujita, Moscarini, and Postel-Vinay, 2019). In this paper, we connect these two macroeconomic trends. We study the effects of a productivity slowdown on firm and worker dynamics in an economy with endogenous growth where the reallocation of labor between old obsolescing firms and new productive firms is intermediated by a frictional labor market. We show that a growth slowdown that originates from weaker imitation —an interpretation of 'ideas getting harder to find'— predicts the above facts. Surprisingly, however, it also generates a countervailing *decline* in labor misallocation.

Our model extends Bilal, Engbom, Mongey, and Violante (2021) (henceforth, BEMV) to study the impact of economic growth on the labor market via creative destruction in the spirit of the seminal contribution by Aghion and Howitt (1994). The BEMV framework is a steady-state model of firm and worker dynamics in frictional labor markets that accommodates both firm turnover via entry and exit in the tradition of Hopenhayn (1992), and worker turnover via job-to-job transitions in the tradition of Postel-Vinay and Robin (2002).

Conceptually, creative destruction occurs along two margins. This first margin is that newly created businesses replace obsolete incumbents. This channel forms the backbone of models in the creative destruction literature and requires well-defined boundaries of the firm. Once firms innovate, they establish a monopolistic position in producing a variety, and face a firm-level marginal revenue function that is strictly decreasing in output, and hence in employment. Firms then hire an optimal amount of labor and firms of different productivities and sizes co-exist.

The second margin of creative destruction is that key factors of production such as labor take time to reallocate to productive newcomers. This channel is mediated by labor market frictions that have remained largely absent from the creative destruction literature. Similarly, the deviation from constant marginal revenue that is common in growth models, is extremely rare in search models. The reason is that it creates conceptual challenges, unresolved until now, in the determination of surplus sharing. The source of the problem is on-the-job search which implies heterogeneity in outside options for workers, depending on their labor market history. As a consequence, solving for firms decisions requires keeping track of the entire distribution of firm wages, a daunting task.

In BEMV we propose a contractual environment that solves this conundrum and so allows for a proper theory of the firm—where firm boundaries are determined by either diminishing returns in technology or consumers' taste for variety—alongside a frictional model of the labor market.[1] We show that all allocations can be derived from the maximization of the *joint surplus* of the firm and its incumbent workers, technically a simple problem, which leads to the emergence of an equilibrium *job ladder in marginal surplus*. In this paper we add growth via imitation in the spirit of Luttmer (2007) and study a balanced growth path equilibrium. Good ideas are easier to find when entrants are better able to imitate ideas in the right tail of incumbent productivity, and harder to find when entrants draw them from the left of the incumbent distribution. In sum, the framework accommodates multi-worker firms, endogenous entry and exit, endogenous productivity growth with creative destruction, job reallocation, churning (excess worker turnover), and unemployment.

We calibrate the model to US data, matching moments on worker and firm dynamics. The process of economic growth requires new productive firms to hire workers away from large less productive incumbents and job losses to unemployment as incumbents slowly fall behind the frontier. We therefore ensure that the model replicates statistics on such job-to-job quits, and employment-to-unemployment separations. The process of economic growth also requires firm entry, to replace incumbents, with stochastic growth of new firms toward their optimal size. We therefore ensure that the model replicates statistics on the entry and exit rates of firms, the distribution of employment across firms by age and size, and the rate of average firm growth over the lifecycle.[2]

We use the calibrated model to assess the impact of worsening in entrants' ability to imitate. Four results stand out. First, growth slows down. Second, the implications of a growth slowdown via this mechanism is consistent with facts related to *firm dynamics*. Employment reallocation falls, as entrants are relatively lower productivity and poach less from incumbents, while incumbents obsolesce at a slower rate. The firm entry rate declines, and the share of employment in old firms increases. Following productivity shocks, firms also hire fewer workers, consistent with Decker, Haltiwanger, Jarmin, and Miranda (2020). As we discuss below, a lower rate of obsolescence reduces misallocation, with workers more efficiently sorted across the productivity distribution of firms. Poaching a worker from another firm to fill a new job is more difficult in a less misallocated labor market so firms expand more slowly following

[1] In BEMV, we write the model with DRS technology. Here we adapt it to a monopolistically competitive environment where decreasing returns is in revenue.

[2] A key step in our analysis is to provide a mapping between the equilibrium conditions of (i) the detrended steady-state of the balanced growth path in our economy with growth and (ii) the steady-state of BEMV. This result allows us to directly use the calibration of BEMV.

a shock to their productivity.

Third, we document the implications for *worker dynamics*. We find that on all dimensions labor market flows slow down: employment-to-unemployment (EU), unemployment-to-employment (UE) and employment-to-employment (EE) transition rates decrease. *Ceteris parabus*, the decline in the UE rate would lead to an increase in unemployment, but the decline in the rate of obsolescence reduces the rate of layoffs. On net, the second force is stronger such that unemployment declines. Compared to canonical matching models without on-the-job search, we find that unemployment is subject to weaker creative-destruction effects associated with faster growth. One important reason is that in our environment, as in the real economy, half of job separations are EE transitions which do not entail an unemployment spell.

Fourth, these observations, combined with a decline in job-to-job mobility, may suggest that misallocation is worse when growth is slower due to a lower intensity of imitation. Interestingly, however, we find the opposite. As growth slows, the misallocation that exists in the economy improves. This is observed in a decline in the dispersion of the marginal product of labor across firms and an increase in the correlation between firm size and productivity. This leads to a higher *level* of output, despite slower growth. We find that this level difference is quantitatively significant, in the sense that it has the potential to offset the negative growth effects of lower imitation.

What is behind this last result? Our model is unique in the class of growth models in that firms operate a single product, start small, and then grow via costly hiring of workers. When the rate of economic growth is high, firms' productivity is quickly slipping behind the productivity of the economy as a whole, and so high productivity firms are *less ambitious* with respect to these expansion plans. High productivity firms no longer grow as large, which aggregates up as a misallocation of labor in the economy.

Our model generically encompasses the three standard channels by which higher growth can affect equilibrium outcomes: (i) higher growth increases the equilibrium *interest rate* which reduces firm values, and firm entry; (ii) higher growth has a *capitalization effect* that increases the incentives to firm entry and vacancy creation due to streams of output being discounted at a lower rate; (iii) because higher growth comes about through higher productivity of new entrants relative to the incumbents, it leads to Schumpeterian *creative destruction* —the hallmark of the Aghion-Howitt approach– which leads to the obsolescence of incumbent firms, as their costs grow faster than their revenues.[3] In addition our model features the new *misallocation* channel that operates via firms being further away from their optimal size when growth is faster. The necessary ingredients for this mechanism to emerge are unique to our model

[3]Because of our preference specification with unitary intertemporal elasticity of substitution, interest rate and capitalization effects exactly cancel out in our calibration.

of economic growth: frictional adjustment and a well defined firm lifecycle. We regard this as a novel economic force, and believe it may be important to study in the future.

1.1 Related Literature

Our paper contributes to a large literature on growth, firm dynamics, and worker flows in frictional labor markets.

Imitation and growth. Our economy falls into the class of growth economies in which the set of varieties produced remains fixed, but the productivity of the technology used in production of these varieties increases over time. The particular way we model imitation is reminiscent of Lucas and Moll (2014), Perla and Tonetti (2014). Rather than microfounding the process of entrants bumping into and imitating incumbents as in these papers, we take a reduced form approach. An exogenous parameter links the thickness of the tail of the tail of entrant productivity draws to the distribution of incumbent productivity, as discussed in Luttmer (2010). We model ideas being are harder to find as a comparative static in which this parameter skews entrants' draws away from the tail of incumbent productivities. Future work may adapt our framework to include expanding varieties, or endogenize this reduced form process via meetings of entrants and incumbents or as spin-offs from incumbent firms. We return on this point in the conclusions.

In this paper we also abstract from horizontal innovation and endogenous improvements in productivity. Both have been combined with imitation in recent papers (Konig, Lorenz, and Zilibotti, 2016; Acemoglu, Aghion, Griffith, and Zilibotti, 2010; Benhabib, Perla, and Tonetti, 2021). The new misallocation channel that we identify will carry over to such models of economic growth. Consider a setup where firms can invest in improving their productivity (as in, for example, Akcigit and Kerr, 2018), and then grow via costly accumulation of labor through frictional labor markets toward their larger optimal size. With lower growth, firms are more ambitious in these expansions plans, and hence invest more in innovation. The same force should, in principle, apply to horizontal innovation. The key ingredient in both cases is that innovation is followed by costly growth of the firm, which is the empirically realistic case.

Growth models with frictionless adjustment. In this class of models (e.g. Garcia-Macia, Hsieh, and Klenow, 2019), firms instantaneously grow to their optimal size and labor is reallocated through a frictionless labor market. Moreover, all changes in employment are due only to creative destruction. Our result that slower growth leads to declining dynamism is reminiscent of Akcigit and Ates (2021) who, in a frictionless model of step-by-step innovation, show that a decline in a parameter that governs exogenous diffusion of technology from leaders to followers can generate falls in productivity growth, entry and job reallocation. Akcigit and Ates (2019) also compute the transition path between balanced growth paths and feature a horse-race between alternative explanations. They find that a slow-down of knowledge diffusion from frontier to laggard firms (and indirectly entrants) can explain a large portion of the observed facts. Relative to this work, we introduce labor market frictions, and hence unemployment and the distinction between worker and job turnover) and uncover the new misallocation channel.

Costless adjustment—i.e., abstracting away from convex or non-convex adjustment costs or costs microfounded through labor market frictions—maintains tractability in all these models by making employment decisions static. In our model firms receive shocks to productivity—or isomorphically, demand—that cause changes in employment, and experience obsolescence due to the creative destruction from entrants that leads them to drift toward exiting the economy. In this sense our model introduces to the creative destruction literature in the tradition of Aghion and Howitt (1992, 1994) features of firm dynamics models with costly adjustment, in the tradition of Hopenhayn and Rogerson (1993), where the costly adjustment in response to demand or productivity shocks occurs through frictional labor markets, in the tradition of Postel-Vinay and Robin (2002).

Search models without growth. We build on the random search set-up developed by Postel-Vinay and Robin (2002) which has become a workhorse of the literature: Bertrand competition between employers for workers and wage renegotiation under mutual consent. Our contribution is to generalize this *sequential auction* protocol to multi-worker firms with diminishing returns in technology and show how one can still solve the model's equilibrium through the notion of joint surplus which yields a dramatic simplification and analytical tractability. In traditional versions of random search models with linear technology the size distribution of firms is either indeterminate or non degenerate only because of the existence of search frictions: as frictions disappear, all workers become employed at the most productive firm, an implausible limiting behavior. Our approach, instead, goes back to (Lucas, 1978): the dominant force that delivers a non-degenerate firm-size distribution is the combination of diminishing returns in production and heterogeneity in productivity.[4] The frictionless limit of our model is, therefore, a version of the competitive firm dynamics model of Hopenhayn (1992).

Growth models with frictional adjustment. Recent papers have microfounded this adjustment process by combining search and growth. Engbom (2017) integrates economic growth into a model with

[4]See also Elsby and Gottfries (2021) for a related framework.

job to job mobility as in Postel-Vinay and Robin (2002). As in other search models with constant returns to scale in production, the unit of analysis is not a firm, but a job. As explained, our model is closer to the tradition in the firm dynamics literature where the unit of analysis is a firm, and its boundaries naturally determined by decreasing returns, here due to monopolistic competition. Martellini and Menzio (2020) show that exogenously declining search frictions in a Diamond-Mortensen-Pissarides model generates a unique balanced growth path equilibrium when productivity is match specific and matches are inspection goods. We keep search frictions constant, and study a balanced growth path that emerges from imitation and creative destruction: newcomers with the best ideas replace obsolete incumbents in the production of one of the varieties consumed by households.

Outline. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses its parameterization. Section 4 presents the results. Section 5 concludes.

2 Model

In this section we first describe the household, government and production sector. Next, we obtain the surplus representation from which all allocations are derived, and illustrate the contractual environment which underlies it. Finally, we explain how to construct a balanced growth path (BGP), and define an equilibrium. The model is written in continuous time.

2.1 Household

The representative household is composed of $\bar{\mathtt{n}}$ individuals who supply inelastically one unit of time to the labor market. The size of the population is constant. A share $\mathtt{u}_t$ of individuals is unemployed and the remaining $\mathtt{n}_t = \bar{\mathtt{n}} - \mathtt{u}_t$ are employed. Employed workers receive wage payments from their firm and unemployed workers receive unemployment benefits from the government. There is full insurance within the family, and thus the household problem can be split into a choice of aggregate consumption and a second stage where the consumption is distributed across household members. This second stage is irrelevant for labor market dynamics, so we abstract from it and focus on the former.

The household discounts the future at rate ρ and is endowed with one unit of a fixed factor each period. It may rent $X_t \leq 1$ units of this factor at rate p_t^X to firms who need it for production. The household gets utility out of consumption C_t, which we assume is log, and linear utility from the unrented stock of the fixed factor.[5] Flow utility is therefore:

$$U(C_t, X_t) = \log C_t + \eta(1 - X_t)$$

Utility from consumption C_t owes to consuming a fixed measure $\mathtt{m}$ of varieties of goods. We assume that C_t is a CES aggregator over these goods with elasticity of substitution $1/(1-\alpha)$:

$$C_t = \left[\int_0^{\mathtt{m}} c_{it}^{\alpha} di\right]^{\frac{1}{\alpha}}.$$

Given a set of prices p_{it} for each good, the household demand for each good can be characterized by the usual demand system:

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\frac{1}{1-\alpha}} C_t, \qquad P_t = \left[\int_0^{\mathtt{m}} p_{it}^{-\frac{\alpha}{1-\alpha}} di\right]^{-\frac{1-\alpha}{\alpha}} \tag{1}$$

The first order conditions with respect to C_t and H_t yield the equilibrium price $p_t^X = \eta P_t C_t$. We assume that the household trades shares of the mutual fund that owns all firms in the economy, and trades a risk-free bond in zero net supply. As is standard, this implies that in equilibrium on a BGP, firms discount future payoffs with a constant risk-free rate $r = \rho + g$.

2.2 Government

Unemployed workers receive benefits b_t from the government that are funded by a constant tax on sales τ. Hence the government budget constraint is:

$$\mathtt{u}_t b_t = \tau \int_0^{\mathtt{m}} p_{it} y_{it} di = \tau P_t Y_t \tag{2}$$

Define $\widetilde{b} = b_t / P_t Y_t$, constant along the BGP, and note that $\widetilde{b} = \widetilde{b}(\tau, \mathtt{u}) = \tau/\mathtt{u}$, since also $\mathtt{u}$ is invariant on the BGP.

[5] This fixed factor can be interpreted in different ways. One example is managerial time needed by the firm to perform certain activities (i.e., entry, operating and recruiting). Another example is land, also needed for the same activities.

2.3 Incumbent firms

Each incumbent firm is a monopolist in producing one of the $\mathtt{m}$ varieties. Firms operate with a linear production function $y_{it} = \overline{z}_{it} n_{it}$. Firm-level productivity $\overline{z}_{it}$ follows a Geometric Brownian motion:

$$d \log \overline{z}_{it} = -\overline{\mu} dt + \overline{\sigma} dW_t \,.$$

In order to remain in operation, a firm incurs a flow fixed cost c_f, and in order to hire workers it has to spend vacancy costs $c_v(v, n)$. Both costs are assumed to be denominated in the fixed factor which the firm rents from the household sector.

Invoking the theoretical results in BEMV, which we summarize in Section 2.5 below, we can solve for the equilibrium allocation of the economy by focusing on the surplus of a coalition between a firm and its n incumbent workers. The flow surplus of such organization is

$$\pi_{it} = (1-\tau) p_{it} \overline{z}_{it} n_{it} - b_t n_{it} - p_t^X c_f - p_t^X c_v (v_{it}, n_{it})$$

where the term $b_t n_{it}$ captures the flow outside options of the incumbent workers. The demand function implies that the price a firm gets for its output of variety i is

$$p_{it} = \left(\frac{y_{it}}{Y_t}\right)^{-(1-\alpha)} P_t$$

Exploiting the fact that $Y_t = Y_0 e^{gt}$ on the BGP, the post-tax revenues of the firm are

$$(1-\tau) p_{it} y_{it} = P_t Y_t \times (1-\tau) \left(\frac{y_{it}}{Y_t}\right)^{\alpha} = P_t Y_t \times Y_0^{-\alpha} \underbrace{(1-\tau) \overline{z}_{it}^{\alpha} e^{-\alpha g t}}_{= z_{it}} n_{it}^{\alpha}$$

where the firm's *relative productivity* $z_{it} = (1-\tau) \overline{z}_{it}^{\alpha} e^{-\alpha g t}$ follows

$$d \log z_{it} = \mu dt + \sigma dW_t \tag{3}$$

where $\mu = -\alpha(\overline{\mu} + g)$ and $\sigma = \alpha \overline{\sigma}$.

Using the government budget constraint, the first order condition $p_t^X = \eta P_t Y_t$, and the equilibrium condition $C_t = Y_t$, flow post-tax surplus can be written:

$$\pi_{it} = P_t Y_t \times \left[Y_0^{-\alpha} \left(z_{it} n_{it}^{\alpha} \right) - \widetilde{b}(\tau, \mathtt{u}) n_{it} - \eta c_f - \eta c(v_{it}, n_{it}) \right] \tag{4}$$

2.4 The labor market

Unemployed and employed workers search in a common, single labor market. Unemployed workers search with an intensity that, without loss of generality, we can normalize to one. Employed workers also search for jobs, but with relative search efficiency χ. Hiring firms and job-seekers meet in a common, single frictional labor market. Search is random.

The total number of meetings is given by the CRS aggregate matching technology $m(\mathtt{s}_t, \mathtt{v}_t)$. Inputs to this function are total vacancies $\mathtt{v}_t$ and total units of search efficiency $\mathtt{s}_t = \mathtt{u}_t + \chi \mathtt{n}_t$. Thus, an unemployed worker meets a firm at rate $\lambda^U(\mathtt{s}_t, \mathtt{v}_t) = m(\mathtt{s}_t, \mathtt{v}_t)/\mathtt{s}_t$. An employed worker meets a firm at rate $\lambda^E(\mathtt{s}_t, \mathtt{v}_t) = \chi\lambda^U(\mathtt{s}_t, \mathtt{v}_t)$. A vacancy meets workers at rate $q_t = m(\mathtt{s}_t, \mathtt{v}_t)/\mathtt{v}_t$. The rates q_t and λ^U_t can be expressed in terms of *market tightness* $\theta_t = (\mathtt{v}_t/\mathtt{s}_t)$. The employed workers can become unemployed either because they choose to quit, the firm lays them off, or exogenously at the constant rate δ.

2.5 Contractual environment

As we explain in detail in BEMV, the contemporaneous presence of random search, on-the-job search and a revenue function decreasing in employment makes the firm problem intractable, i.e. computing optimal layoff, retention, and vacancy policies requires keeping track of the entire wage distribution (possibly hundreds or thousands of states).

We propose a minimal set of assumptions on the contractual environment such that the state vector becomes manageable. Three assumptions on bargaining and surplus sharing are common to many single-worker firm environments: (i) lack of commitment; (ii) wage contract renegotiation by mutual consent (i.e. only when one of the parties has a credible threat, or an outside option more valuable than the current contract); (iii) Bertrand competition among employers for employed jobseekers with take-it-or-leave-it offers. For example, these are the assumptions in Postel-Vinay and Robin (2002). Two further assumptions are required in our new multi-worker firm environment: (iv) no value is lost in internal wage renegotiations between a firm and its incumbent workers; and (v) vacancy policies maximize combined firm and worker value –for which in BEMV we offer an explicit microfoundation. Under these assumptions, firm and workers' decisions are privately efficient, as if the firm and incumbent workers maximize their total joint value. The state variables of the joint value (or surplus) function are only two: firm size n and productivity z.

We note that this joint surplus representation uniquely pins down allocations (firm and worker dynamics), but is consistent with multiple wage setting mechanisms that determine how this joint value is split between the parties. Wages, therefore, are not allocative. In order to study the model's implication for wage dynamics, one would have to make additional assumptions. Since the subject of this paper is labor reallocation, we refrain from making extra assumptions and abstract from discussing implications for the wage distribution. For additional details on the contractual environment, we refer directly to BEMV.

2.6 Surplus representation

Let $S(z,n,t)$ denote the joint surplus of an organization composed by a firm —the owner of the technology with productivity z— and by its n workers at time t. Using our expression for the flow surplus (4), the joint surplus is given by the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} rS(z,n,t) - \frac{\partial S(z,n,t)}{\partial t} = \; & \max_{v \geq 0} P_t Y_t \times \left[Y_0^{-\alpha} (z n^{\alpha}) - \widetilde{b}(\tau, \mathtt{u})\, n - \eta c_f - \eta c_v (v, n) \right] \\ & - \delta S_n (z,n,t) \\ & + q v(\theta) \phi S_n (z,n,t) \\ & + q v(\theta)(1-\phi) \int_0^{S_n(z,n,t)} S_n(z,n,t) - S_n(z',n',t) \; dH_t(z',n') \\ & + \mu(z) S_z(z,n,t) + \frac{\sigma(z)^2}{2} S_{zz}(z,n,t). \end{aligned} \tag{5}$$

where $\phi = \mathtt{u}/\mathtt{s}$ is the share of unemployed job seekers and H_t is the employment distribution.

The first line of this equation is the flow surplus. The other lines represent events that can occur to the organization. In the second line, the firm exogenously loses one of its n workers to unemployment at rate δ and as a result it loses the marginal surplus contributed by the lost worker. In the third line, the firm is hiring and meets an unemployed worker who brings marginal surplus $S_n(z,n,t)$ to the coalition. The firm also hires from other firms by poaching (fourth line). Workers at other firms are met according to the employment-weighted distribution of productivity and size, H_n. Upon hiring, total surplus increases by $S_n(z,n,t) - S_n(z',n',t)$. The first term is the gain in value to the firm and incumbent workers due to the new hire. The second term is the value pledged to the new worker, which is equal to the highest value its former employer would pay to retain them. Poaching is successful if this difference is positive and workers flow to the highest marginal surplus firm. Thus the model implies a job ladder in *endogenous marginal surplus*, as opposed to exogenous productivity z as in the canonical Postel-Vinay and Robin (2002) model. Conversely, an incumbent worker may quit to a higher marginal surplus firm. The firm and remaining workers will lose $S_n(z,n,t)$ and so are prepared to increase the worker's surplus by at most $S_n(z,n,t)$ to retain them. Knowing this, the external firm hires the worker by offering them exactly $S_n(z,n,t)$. The joint surplus of the firm, remaining workers and poached worker are therefore unchanged and, as in Postel-Vinay and Robin (2002), no *'EE Quit'* term appears in (5).

The first order condition for the firm's vacancy decision gives

$$c_v(v;z,n) = q(\theta_t)\left[\phi_t S_n(z,n,t) + (1-\phi_t)\int_0^{S_n(z,n,t)} \Big(S_n(z,n,t) - S_n(z',n',t)\Big)\, dH_n\left(S_n(z',n',t)\right)\right] \quad (6)$$

or the marginal cost of hiring (left hand side) equals the expected return from hiring (right hand side). On the *intensive margin*, an increase in S_n increases the return to hiring an unemployed or employed worker one-for-one. On the *extensive margin*, an increase in S_n widens the set of firms from which the firm will poach, increasing the probability of a hire by $(1-\phi)h_n(S_n)$, but hiring from these additional firms yields zero additional value as the target firm's marginal surplus associated with the worker is close to that of the poaching firm.

Finally, at every t, the operation of the firm requires (z,n) to be interior to an *exit boundary*, and an additional *layoff boundary* determines when separations occur:

$$\textit{Exit boundary:} \quad S(z,n,t) \geq 0, \quad , \quad \textit{Layoff boundary:} \quad S_n(z,n,t) \geq 0. \quad (7)$$

The exit boundary states that the joint surplus of the organization must be weakly positive to continue operations. The layoff boundary requires the surplus of the marginal worker to be weakly positive. If it is negative, the firm will instantaneously shed enough workers to restore equality.

2.7 Balanced growth path

We are interested in constructing a balanced growth path equilibrium of an economy in which output Y_t and consumption C_t grow at a constant rate g, and unemployment $\mathtt{u}_t$ is constant. We normalize the aggregate price level P_t in each period to 1. With constant population $\bar{\mathtt{n}}$, employment $\mathtt{n} = \bar{\mathtt{n}} - \mathtt{u}$ is constant on the BGP, and thus average firm size $\mathtt{n}/\mathtt{m}$ is also constant.[6]

It is straightforward to guess and verify that $S(z,n,t) = P_t Y_t S(z,n)$. Since, without loss of generality, we normalize $P_t = 1$ and $Y_t = Y_0 e^{gt}$, then $S(z,n,t) = Y_0 e^{gt} S(z,n)$. Hence $\partial S(z,n,t)/\partial t = gS(z,n,t)$.

[6] As in Luttmer (2007) we could include population growth at a constant rate which would require the number of firms, and varieties, $\mathtt{m}$ to also grow at a constant rate since average firm size must be constant on any BGP.

Using this, along with $r = \rho + g$, we obtain:

$$\rho S(z,n) = \max_{v} \left\{ Y_0^{-\alpha} z n^{\alpha} - \widetilde{b}(\tau, \mathtt{u})\, n - \eta c_f - \eta c(v,n) - \delta S_n(z,n) \right. \tag{8}$$
$$\left. + qv\left(\phi S_n(z,n) + (1-\phi) \int_0^{S_n(z,n)} \left(S_n(z,n) - S(z',n') \right) dH(z',n') \right) \right\}$$
$$+ \mu(z) S_z(z,n) + \frac{\sigma(z)^2}{2} S_{zz}(z,n)$$

The surplus (8) is exactly that in BEMV, with the following two additions: (i) an endogenous constant $Y_0^{-\alpha}$, and (ii) an endogenous equilibrium flow value of unemployment $\widetilde{b} = \tau / \mathtt{u}$.

Note that the drift in relative productivity $\mu(z) = -[\alpha(\overline{\mu} + g) + \sigma^2/2]z$ encodes the fact that growth leads the costs of inputs relative to sales to increase—an *obsolescence (or creative destruction) effect*.[7] As we explain next, the endogenous growth rate of the economy g is determined by the free entry condition.

2.8 Entry

To close the model, it remains to specify a process for firm entry. An entrant firm replaces one of the exiting incumbents in producing one of the $\mathtt{m}$ varieties.

We assume that new firms enter with a relative productivity z that is drawn from a Pareto distribution with some endogenous tail index ζ. Given the productivity distribution of entrants and the law of motion for incumbent productivity (3), it follows from standard arguments (see, e.g. Luttmer, 2007) that the distribution of incumbent productivity is asymptotically Pareto with tail index $2\mu/\sigma^2$. Let the parameter ψ govern how well entrants can replicate the tail of the distribution of incumbent firms: $\zeta = \psi\left(2\mu/\sigma^2\right)$. A *lower value* of ψ means that entrants draw productivities that are further into the tail of the distribution of incumbent firm productivities (i.e., *better imitation*). This is the key parameter that we will vary in our counter-factual experiments.

Entrants are assumed to pay an entry cost c_0, again denoted in units of the fixed factor as all other operating costs, in order to make a productivity draw and start producing with n_0 initial employees. Free-entry then implies the equilibrium condition[8]

$$\eta c_0 = \int S(n_0, z)\, dF(z; \zeta) \tag{9}$$

[7] Because of log utility, interest rate and capitalization effects offset each other exactly and g does not show up in discounting.

[8] Specifically, $p_t^X c_0 = \int S(n_0, z, t)\, dF(z;\zeta)$ which implies $\eta P_t Y_t c_0 = \int P_t Y_t S(n_0, z)\, dF(z;\zeta)$, and condition (9).

where F is the endogenous productivity distribution of entrants. This condition uniquely determines the equilibrium growth rate of the economy. Surplus (8) is decreasing in g, since higher growth increases the rate at which the price of the fixed factor increases relative to the price of the varieties produced. Intuitively, consider a scenario where at a given growth rate, the free entry condition did not hold, i.e. there was a positive expected surplus (net of the entry cost) from entering. More firms would choose to pay the cost and draw new values of productivity. Through imitation, this would generate additional economic growth via an increase in average productivity. Higher growth causes the value of entry to fall until the free-entry condition holds.

2.9 Equilibrium

The equilibrium of the model determines the rate of growth g, and the level of output Y_0 and benefits $\widetilde{b}(\tau, \mathtt{u})$ which enter the detrended HJB equation (8).

First, the free entry condition (9) uniquely determines the equilibrium growth rate of the economy. Second, the level of output Y_0 must be consistent with the production of the heterogenous firms in the economy. Aggregation implies that $\int_0^{\mathtt{m}} p_{it} y_{it} di = P_t Y_t$. Using our expression for relative sales:

$$1 = \int_0^{\mathtt{m}} \frac{p_{it} y_{it}}{P_t Y_t} di = \int_0^{\mathtt{m}} \left(z_{it} n_{it}^{\alpha}\right) \times \left(Y_0^{-\alpha}\right) di \quad , \qquad Y_0^{\alpha} = \mathtt{m} \int_{N \times Z} z n^{\alpha} dH\left(z, n\right).$$

This pins down the initial level of output, which then grows at rate g: $Y_t = e^{gt} Y_0$. Third, we also require $\widetilde{b}\left(\tau, \mathtt{u}\right) = \tau / \mathtt{u}$.

Finally, given an optimal employment policy of firms and the stochastic process for relative productivity, a standard KFE characterizes the distribution of firms over size and relative productivity. We provide the complete definition of the BGP equilibrium in Appendix A.1. Most of the definition follows directly from BEMV with the additional features to accommodate growth, such as those just described.

3 Calibration

The model period is set to a month.[9] For calibration, we use 2011-2016 as our reference period to construct all the empirical counterparts of the model's moments. We make the same functional form assumptions as in BEMV. The matching function is Cobb-Douglas with vacancy elasticity β and match efficiency A, i.e. $A \mathtt{v}^{\beta} \mathtt{s}^{1-\beta}$. The vacancy cost function is $c_v\left(v_{it}, n_{it}\right) = \frac{\bar{c}_v}{1+\gamma}\left(\frac{v_{it}}{n_{it}}\right)^{\gamma} v_{it}$.

[9] Because the model is written in continuous time, we can accommodate any observed data frequency with the appropriate time aggregation within the model.

Parameters in $\boldsymbol{\theta}_{BEMV}$		Value	Moment	Data	Model
			A. Externally set/normalized parameters		
ρ	Discount rate	0.004	5% annual real interest rate		
c_f	Fixed cost of operation	1	Normalization		
$c_v/(1+\gamma)$	Scalar in the cost of vacancies	100	Normalization		
η	Preference parameter	1	Normalization		
n_0	Size of entrants	1	Normalization		
β	Elasticity of matches w.r.t. vacancies	0.5	Petrongolo and Pissarides (2001)		
			B. Estimated offline		
$\mathtt{m}$	Number of active firms	0.043	Average firm size (BDS)	23.340	20.851
γ	Vacancy cost elasticity	3.450	Vacancy filling rate vs. hiring rate	3.450	3.450
d	Exogenous exit rate	0.002	Exit rate, 1000–2499 empl. firms	0.002	0.002
			C. Internally by minimum distance		
μ	Drift of productivity	-0.001	Exit rate (annual)	0.076	0.076
σ	St.d of productivity shocks	0.016	St.d. of log empl. growth (annual)	0.420	0.354
α	Curvature of production	0.817	Empl. share of 500+ firms	0.518	0.527
ζ	Shape of entry distribution	11.844	JC rate, age 1 firms (annual)	0.247	0.255
A	Matching efficiency	0.195	Nonemployment rate	0.100	0.100
ξ	Relative search efficiency of employed	0.151	EE rate (quarterly)	0.048	0.041
δ	Exogenous separation rate	0.017	EN rate (quarterly)	0.056	0.055
$\widetilde{b}$	Transformed flow value of leisure	1.029	JD rate of incumbents (annual)	0.092	0.093

Table 1: Estimated parameters and targeted moments

Notes: Annual firm dynamics moments are from HP-filtered Census BDS data between 2011–2016, with the exception of the standard deviation of annual growth rates, which is from Elsby and Michaels (2013). Quarterly worker flows are from HP-filtered Census J2J data between 2011–2016.

Some parameters can be set externally. As we discuss in BEMV, the parameters $\bar{c}_v$ and c_f can be normalized. Since the preference parameter η multiplies all costs in the model we can also normalize η. We also set the number of workers at new firm n_0 to 1 (the entrepreneur) and β, the elasticity of the matching function with respect to vacancies, to 0.5.

Moreover, we can estimate some of the model parameters outside the model. The measure of varieties and firms in the economy $\mathtt{m}$ is fixed on the balanced growth path. We choose an entry cost c_0 to deliver a value of $\mathtt{m}$ that is consistent with an average firm size of 23, which is fairly stable over time in the US.[10]

To estimate the elasticity of the vacancy cost function γ, we build on the empirical finding of Davis, Faberman, and Haltiwanger (2013) who, from JOLTS microdata, document a nearly log-linear empirical relationship between the vacancy rate and the vacancy filling rate. We show that our model has a similar prediction. Starting from the firm's optimal vacancy policy and using a log linear approximation we obtain:

$$\log \frac{v(z,n)}{n} \approx \kappa_0 + \kappa_1 \log\left(\frac{h(z,n)}{v} - \kappa_2\right), \quad \text{with} \quad \gamma = \frac{1}{\kappa_1} \tag{10}$$

where κ's are combinations of structural parameters, and h denotes hires. Firms with high marginal surplus post more vacancies per worker, and fill them more quickly as they can poach labor from more

[10]When calibrating the model, note that one can compute the value of entry under any fixed number of firms $\mathtt{m}$ and then set c_0 ex-post appropriately such that the free-entry condition holds. Therefore, assuming the calibration hits the target for the employment rate (0.90), $\mathtt{m}$ can be chosen offline to match the average firm size. When computing comparative static exercises, we fix $\mathtt{m}$ and c_0. The firm size distribution, and average size, endogenously change.

firms. We compute these objects in JOLTS microdata in narrow monthly growth rate bins, then estimate (10) by non-linear least squares.[11] Our estimates imply $\gamma = 3.45$.

Finally, we add to the model a rate of exogenous firm exit d, to match the exit of large firms in the data.

This approach leaves the following parameters to calibrate internally: $\boldsymbol{\theta}_{BEMV} = \{\mu, \sigma, \zeta, \alpha, A, \chi, \delta, \widetilde{b}\}$. These parameters are estimated jointly by minimum distance. Although the estimation is joint, we heuristically discuss what moments particularly inform what parameter. The overall drift μ, is informed by the rate of firm exit. The standard deviation of productivity shocks σ, is informed by the standard deviation of annual log employment growth. The thickness of the tail of entrant productivity draws ζ, is informed by job creation among young firms. The diminishing return parameter α is informed by the share of large firms. Matching efficiency A, is informed by the non-employment rate and relative search efficiency of employed workers χ and exogenous separation rate δ are informed by EE and EN rates.[12]

Finally, the flow value of non-employment $\widetilde{b}$—which in BEMV is a preference parameter, instead of being financed by taxes—is informed by the rate of job destruction of incumbent firms. Once $\widetilde{b}$ is set, we determine τ that balances the government budget. We keep τ constant across comparative statics. Table 1 summarizes these moments and shows that the model does well in matching them. We note that our estimate of α implies an elasticity of substitution across varieties of 5.46. This value would imply a markup of about 20 percent in a standard monopolistically competitive model.

The underlying parameters of our new endogenous growth model are $\boldsymbol{\theta}_{Growth} = \{\overline{\mu}, \overline{\sigma}, \psi, \varphi\}$. Given the estimated parameters of $\boldsymbol{\theta}_{BEMV}$ in Table 1, we can obtain $\boldsymbol{\theta}_{Growth}$ as follows. First, we set a growth rate of the economy of 1.75 percent annually, which delivers g. Second, we back out $(\overline{\mu}, \overline{\sigma})$ via the mapping

$$\overline{\mu} = -\left(\frac{\mu}{\alpha} + g\right) \quad , \quad \overline{\sigma} = \frac{\sigma}{\alpha}.$$

Third, given $\{\mu, \sigma, \zeta\}$ we back out the implied ψ given our model of imitation: $\psi = \zeta\sigma^2/2\mu = 1.24$. Fourth, given α we back out the elasticity of substitution $\varphi = (1-\alpha)/\alpha$, which delivers $\varphi = 4.46$. This value would imply a markup of about 30 percent in a standard monopolistically competitive model.

[11] We use updated estimates of this relationship from BLS microdata from Mongey and Violante (2019).

[12] As in BEMV, we use a broader definition of the pool of non-employed job-seekers than in the standard unemployment definition of the BLS. This accounts for the fact that a significant number of hires come directly from out of the labor force and some of our data sources (JOLTS and Census J2J) do not identify whether the origin of hires or destination of separations is unemployment or non-participation. In particular, our definition of the non-employment rate is constructed as follows. The numerator equals the sum of the unemployed (FRED series UNEMPLOY) plus those out-of-the-labor-force who answer that they ‘currently want a job’ in the CPS (NILFWJN). The denominator equals the sum of the civilian labor force (CLF16OV) plus the same subgroup of those out-of-the-labor- force (NILFWJN). From 2011-2016 this ratio is, on average, just above 10 percent.

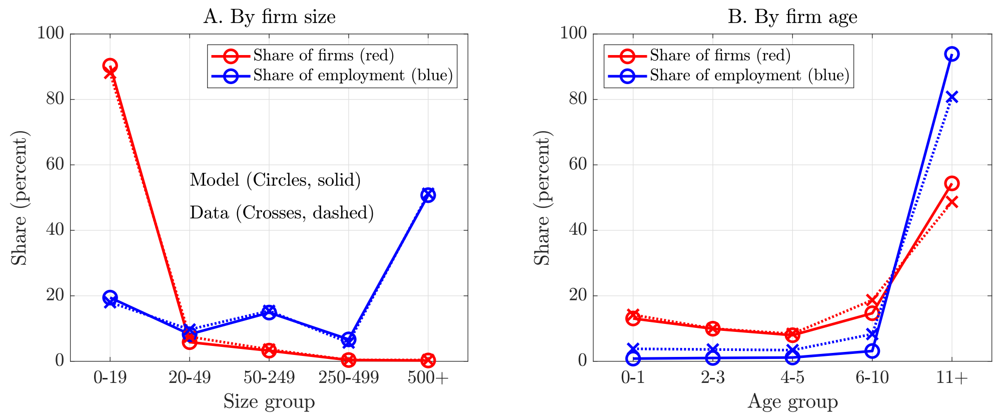

Figure 1: Distribution of firms and employment by firm age and size in data and model. Source: Census BDS.

Figure 2: The model's firm distribution in the space of productivity z and size n at different point of the life cycle.

Summary and non-targeted moments. In summary, we have calibrated a model of endogenous growth at rate g that matches key facts regarding firm and worker dynamics in the US economy over the period 2011-2016.

In BEMV we show that the model matches a wide array of non-targeted moments. Here we focus on lifecycle firm dynamics, which are key to our results regarding misallocation and economic growth. First, Figure 1 verifies that the model matches the empirical distribution of firms and employment by firm age and firm size. Most firms are small, but most employment is at large firms. Most firms are old and most employment is at old firms. Second, Figure 2 reveals the dynamics of the distribution of firm size and productivity over the lifecycle that deliver the empirical marginal distributions in Figure 1. The figure shows how misallocation is partially resolved over the lifecycle, with employment and productivity becoming increasingly correlated as firms age. A key feature of the model, which we highlight in BEMV, is that the model can generate small, productive, young firms. Young firms with high productivity are small because they are young and yet to accumulate workers through the frictional labor market. This is a key source of misallocation in the economy, due to search frictions, to which we will return when we discuss our results.

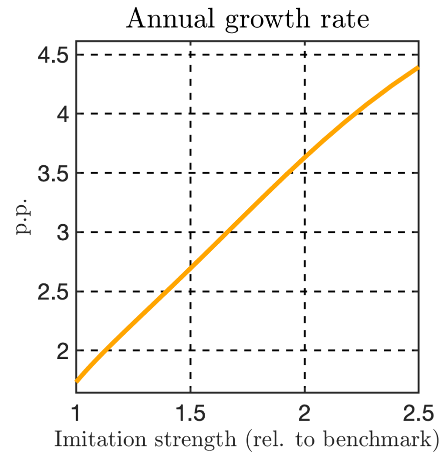

Figure 3: Imitation strength ψ and growth rate g

Notes: The baseline value of the imitation parameter is $\psi_0 = 1.24$, and the baseline rate of growth is $g_0 = 1.75$ percent. A lower value of ψ implies that the tail of the entrant productivity distribution is *fatter*. Relative imitation strength is therefore measured as ψ_0/ψ'.

4 Implications of slower growth for business dynamism, labor market dynamics and misallocation

Our key counterfactual exercise is a comparative static change in the imitation parameter ψ across BGPs, holding all other parameters fixed.[13] Figure 3 plots the rate of growth as we vary ψ. We plot the relative imitation strength which we define as ψ/ψ', since smaller values of ψ' imply draws of productivity that are further skewed toward the right of the incumbent productivity distribution. As we would expect, a lower ψ' raises the aggregate growth rate.

In the rest of this section, we analyze the implications of a change in the growth rate of the economy for business dynamism, labor market dynamics and misallocation. Since we estimated the model to 2011–2016, our thought experiment is going backward in time or making ideas easier to find relative to the present (a fall in ψ and a rise in g).

4.1 Business dynamism

Figure 4 shows firm dynamics outcomes as we vary the underlying imitation parameter ψ. Since the parameter is not of direct interest, we plot these key outcomes as a function of the resulting endogenous growth rate.

Not surprisingly, a fall in ψ (i.e. moving to the right in these figures) is associated with an increase in

[13]In the Appendix, we discuss how to solve such a counterfactual economy.

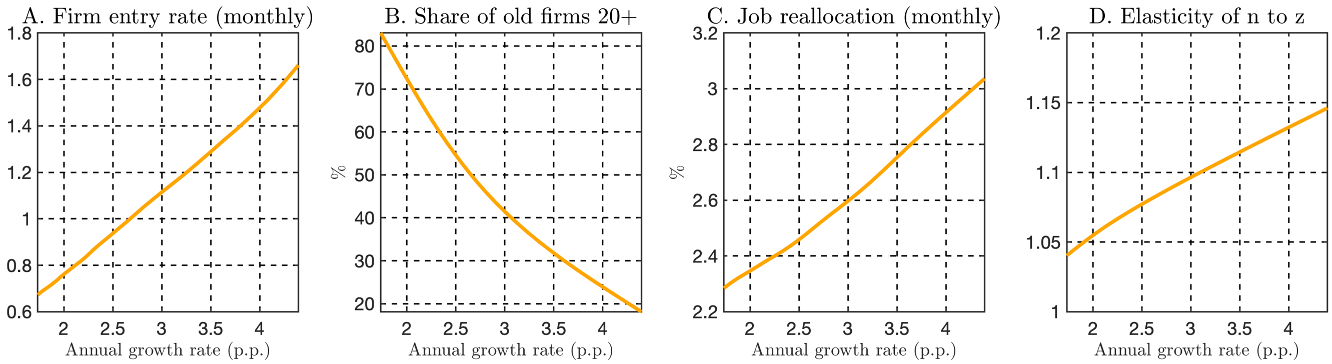

Figure 4: Business dynamism as ideas get harder to find

Notes This figure plots moments from counterfactual BGP equilibria. We plot each moment against the growth rate of the economy, where variation in the growth rate is due to changes in the strength of imitation (Figure 3).

firm creation (panel A). As imitation from the incumbents is easier, entrants are relatively more productive and firm creation is encouraged. As more entrants attempt to enter and there is more competition for inputs, the price of inputs rises faster. Consequently, incumbent firms exit at a faster pace and the rate of obsolescence increases. This is our counterpart of the *creative-destruction effect* of Aghion and Howitt (1994). Quantitatively, a fall in the aggregate growth rate of one percentage point annually—broadly consistent with the US experience over the past decades—is associated with a decline in firm entry of about 30 percent. For comparison, the US has experienced a reduction in firm entry of over 40 percent since the early 1980s to now (Pugsley and Sahin, 2019; Akcigit and Ates, 2021).

Also consistent with the US data over the past 40 years, a larger share of firms are old when growth is slower (panel B). This outcome arises naturally in our model in response to a fall in the rate of obsolescence—firms remain in the market for longer, extending the firm life-cycle. The overall job reallocation rate falls with the aggregate growth rate (panel C). The reason is that as the rate of obsolescence declines, firms do not fall behind the market as quickly. Consequently, there is less need to reallocate employment across production units. Quantitatively, as the aggregate growth rate declines by 1 percentage points annually, the job reallocation rate decreases by around 10 percent. For comparison, the US has experienced a decline in job reallocation of roughly 30 percent since the early 1980s to now (Akcigit and Ates, 2021).

As pointed out by Decker, Haltiwanger, Jarmin, and Miranda (2020), reallocation can decline either due to lower dispersion of idiosyncratic shocks faced by businesses, or to weaker marginal responsiveness of firms to shocks. They show that it is the responsiveness of business-level employment to productivity that has weakened. Panel (D) of Figure 4 shows that the model predicts a decline in the elasticity of employment to productivity, as growth slows down. Quantitatively, the decline is smaller than its

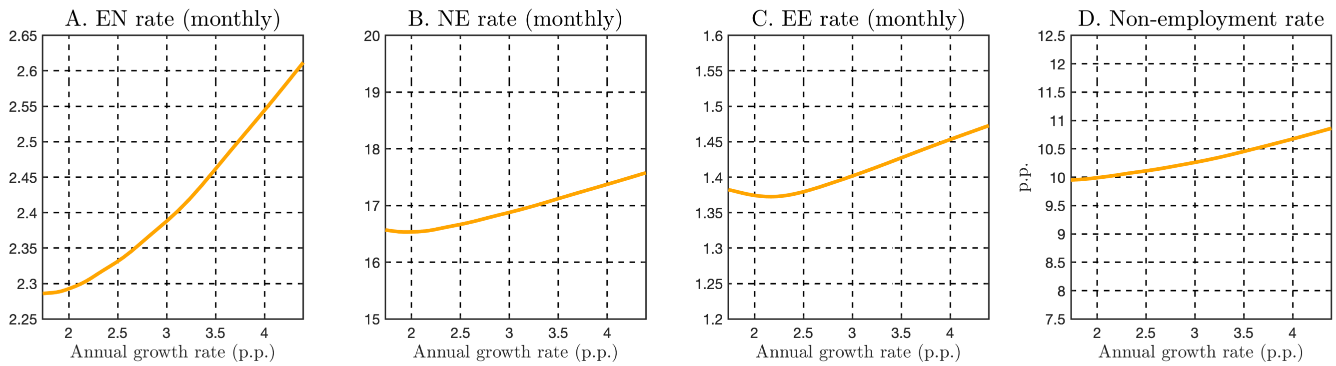

Figure 5: Labor market flows as ideas get harder to find

Notes This figure plots moments from counterfactual BGP equilibria. We plot each moment against the growth rate of the economy, where variation in the growth rate is due to changes in the strength of imitation (Figure 3).

empirical counterpart, but the pattern is qualitatively consistent. A key result, which we discuss below, is that workers are better allocated across firms at lower levels of economic growth, despite lower rates of job reallocation. This makes growing following a positive productivity shock more costly, as it becomes less likely that a posted vacancy will be matched with a worker at a lower productivity firm. This novel mechanism links job-to-job mobility, growth, and the responsiveness of firm-level employment to idiosyncratic shocks.

4.2 Labor market flows

Figure 5 plots worker flows as a function of the aggregate growth rate which, in turn, differs across BGP's due to differences in the imitation parameter ψ. As ideas get harder to find— moving from right to left in these figures—worker reallocation rates fall. As the rate of obsolescence decelerates, firms turn over more slowly in the relative productivity distribution. Consequently, there is less need to reallocate employment, which shows up as lower worker flows.

The pattern of lower worker flows is particularly evident in the EN rate and less pronounced for the EE rate. In our model, firms shrink either through quits or layoffs. When a firms' relative productivity is high, it only shrinks via quits. As its relative productivity falls, due to negative drift and obsolescence, the rate of quits increases. Finally, the firm starts to layoff workers in addition to quits, as the marginal value of a worker $S_n(z, n)$ hits the value of unemployment. With a lower rate of obsolescence, relative productivity drifts downward more slowly, extending the duration of time the firm is away from the layoff boundary. This significantly reduces EN layoffs.

The NE rate also falls with slower growth, but by less than the separation (EN) rate, and as a result the non-employment rate falls slightly. This is not surprising, since the capitalization effect is neutralized by

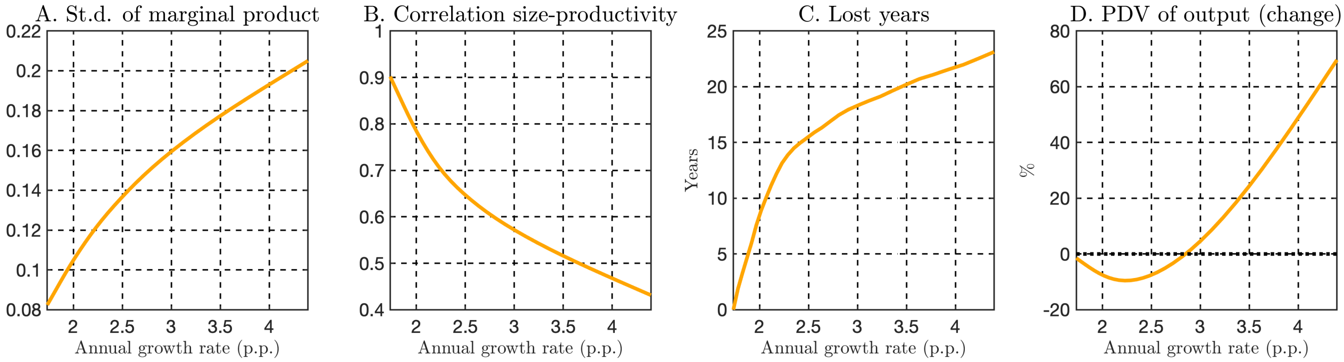

Figure 6: Improving misallocation as ideas get harder to find

Notes This figure plots moments from counterfactual BGP equilibria. We plot each moment against the growth rate of the economy, where variation in the growth rate is due to changes in the strength of imitation (Figure 3).

the interest rate effect in general equilibrium with log utility (see e.g., Aghion and Howitt, 1994).[14] Note, however, how stable the non-employment rate is in our model as the growth rate in the model almost triples. This result stands in sharp contrast with the predictions of plausibly calibrated canonical matching models featuring 1-worker 1-job matches and linear utility, where the effect of creative-destruction on the unemployment rate is very strong (Pissarides and Vallanti, 2007).

A first departure of our exercise from that in Pissarides and Vallanti (2007) is that in our case the increase in g is *endogenous*, caused by an increase in entrant imitation. With more imitation, firms are more productive, grow more quickly, and this reduces unemployment. A second departure is the inclusion of job-to-job transitions (as in Michau, 2013). With job-to-job flows, the reallocation due to creative destruction can occur without generating the same amount of unemployment. Hence, the mechanism underlying the change in g in a search and matching model, as well as the model environment, affect the sensitivity of unemployment to growth.

4.3 Misallocation

Figure 6 illustrates a surprising result. As growth slows, the misallocation that exists in the economy improves, despite there being less firm and worker turnover. The dispersion of the marginal product of labor across firms falls (panel A) and the correlation between size and productivity increases (panel

[14]With log utility, the equilibrium discount rate in the detrended HJB equation (8) is ρ. If we had instead assumed CRRA preferences with a relative risk aversion γ, then the effective discount rate in (8) would be $\rho + (\gamma - 1)g$. If $\gamma < 1$, then the capitalization effect would resurface and would increase the relative value of future payoffs, increasing investment in creating jobs in terms of both the extensive margin of entry and intensive margin of vacancy creation by incumbents. Hence higher g, would lead to lower unemployment. An interesting avenue for future research would be understanding the strength of this capitalization effect in models like ours where firms, rather than matches, are the relevant unit. Individual job matches are relatively short-lived (4-5 years), which leads to a weak capitalization effect in existing work (Pissarides and Vallanti, 2007), whereas the average age of a firm in the US is closer to 20 years.

B). This is consistent with the well documented reallocation of sales in the economy toward larger firms (Autor, Dorn, Katz, Patterson, and Van Reenen, 2017), and leads to a higher *level* of output, despite slower long-run growth.

What explains this finding? Our model is unique in the class of growth models in that firms operate a single product, start small, and then grow via costly hiring of workers. When the rate of economic growth is high, firms' productivity is quickly slipping behind the productivity of the economy as a whole. Therefore, the horizon of firms shrinks. All firms become *less ambitious* with respect to their expansion plans. Hiring is costly and takes time, and has a lower benefit when the incumbent firm will be swiftly replaced by better firms. A given reduction in firms' expected horizon of operations is, however, relatively more costly for high productivity firms, who forgo a higher stream of profits when they exit. Thus, high productivity firms no longer grow as large, which shows up as increased misallocation of labor in the economy.

We find that this output loss from misallocation can be quantitatively important. Panel (C) computes a measure of 'lost years' caused by this level effect. It asks how many years, at the baseline annual growth rate of 1.75 percent, it would take to compensate the downward shift in the level of output caused by misallocation. For example, the additional misallocation that occurs at a growth rate of 4 percent, would be offset by 22 years of economic growth at 1.75 percent. The relation is highly nonlinear: when the economy grows fast, the growth-induced misallocation is weak. Starting from low growth rates, however, a rise in the growth rate of 1 percentage point per year would require over 15 years of baseline-growth to make up for the productivity loss from misallocation.

Panel (D) shows that in terms of the present discounted value of output, an improvement in imitation that increases the growth rate to at least 2.9 percent is required to offset the misallocation effects from higher growth. Admittedly, here we do not compute the transition between one balanced growth path and the other, and ψ is an exogenous parameter. Nonetheless, these calculations suggest that computing the misallocation costs of higher growth may be an important consideration in the formulation of pro-growth economic policies that could be implemented to change objects related to diffusion of ideas in the economy.

Taking stock, while a growth slowdown can have severe implications for business dynamism, the silver lining is that it can limit the degree of misallocation induced by labor market frictions. To the best of our knowledge this is a new mechanism, with the necessary ingredients being the frictional adjustment of small entering firms toward their optimal size. Our model has these features and also quantitatively replicates the joint age and size distribution of firms in the U.S. economy, giving us some confidence in our conclusion that this force may be quantitatively relevant.

5 Conclusions

This paper has evaluated the impact of slowing economic growth on labor market dynamism and misallocation. To that end, we have proposed a tractable extension of the rich framework of firm and worker dynamics in Bilal, Engbom, Mongey, and Violante (2021) that incorporates endogenous growth via creative-destruction. Entrants imitate incumbents, gradually pushing out the technological frontier. Employment reallocates from old, obsolescing incumbents to new, more productive entrants. Although this process takes time due to labor market frictions, it does not necessitate workers spending time in unemployment. Instead, a large share of this reallocation takes the form of direct job-to-job moves, consistent with recent empirical evidence.

We apply the framework to show that a fall in entrants' ability to imitate incumbents, which captures the view that 'ideas are getting harder to find' (Bloom, Jones, van Reenen, and Webb, 2020) accounts for a range of US labor market patterns over the past 40 years. Firm creation naturally declines, as potential entrants are discouraged by the fact that coming up with good business ideas is more difficult. As fewer firms enter, less competition for inputs reduces the rate at which prices of factors of production rise. Consequently, incumbent firms become obsolete at a slower pace. Firm exit falls, the firm life-cycle lengthens, job reallocation declines, as do rates of job loss and job-to-job mobility, all consistent with secular trends in the US. The lower entry rate and the lower productivity of entrants both contribute to lower the aggregate growth rate. At the same time, because growth is creative-destructive, allocative efficiency *improves*.

As a framework that links growth and labor market outcomes in a micro-founded theory of firm and worker dynamics, our model offers several promising directions for future research. For instance, it would be valuable to incorporate endogenous incumbent innovation, given recent evidence of the importance of such innovation for growth (Akcigit and Kerr, 2018): our model only features (a reduced form) exogenous incumbent innovation through improvements in productivity. Such an expanded framework may provide a richer understanding of how different sources of growth impact the labor market. Moreover, it would be interesting to further endogenize the process through which entrants learn about and build upon incumbent firms' technologies, along the lines of Chatterjee and Rossi-Hansberg (2012) or Akcigit, Celik, and Greenwood (2016) where new ideas occur to individual people, not organizations. A natural hypothesis is that this process involves the founder working at established firms, before spinning off to start a new firm. Under this view, the working of the labor market may affect the flow of knowledge in the economy.

A full policy analysis would require a characterization of constrained efficient allocations which is beyond the scope of this paper. Our results, however, do suggest that any macro or industrial policies that promote faster growth must take into account the 'unintended consequence' of the higher labor misallocation and its impact on productivity. In addition, the faster labor reallocation, and the implied higher earnings volatility, could have adverse effects effect on workers' welfare in the presence of risk aversion and imperfect risk sharing. Interestingly though, our model with on-the-job-search also indicates that the ability of workers to move directly from one job to another may mitigate such unfavorable effects of creative-destructive growth on workers' welfare, since reallocation does not necessarily require workers to become unemployed in the process, as in the first generation of 'growth and unemployment' models. A richer assessment of the welfare consequences of growth across the distribution of workers is best left for future work.

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APPENDIX

A Additional details

A.1 Equilibrium

A *balanced growth path equilibrium* with positive entry consists of: (i) a detrended joint surplus function $S(z,n)$; (ii) a vacancy policy $v(z,n)$; (iii) a law of motion for firm level employment $\frac{dn}{dt}(z,n)$; (iv) a stationary distribution of firms $H(z,n)$; (v) vacancy- and employment-weighted distributions of marginal surplus $H_v(S_n)$ and $H_n(S_n)$; (vi) a positive mass of entrants $\mathtt{m}_0$, (vii) a vacancy meeting rate $q(\theta)$ and conditional probability of meeting an unemployed worker ϕ, (viii) an initial level of output Y_0, unemployment rate $\mathtt{u}$, rate of growth g, rate of return r and price of the fixed factor p_t^X, (ix) prices p_{it} and quantities of goods y_{it}. Under the normalization $P_t = 1$, these objects satisfy:

(i) Total surplus $S(z,n)$ satisfies the detrended HJB equation (8) under $(Y_0, \mathtt{u}, g)$—and associated boundary conditions. That is $\mu(z) = -[\alpha(\overline{\mu} + g) + \sigma^2/2]z$, the discount rate is $r = \rho + g$, the detrended rate of benefits is $\widetilde{b} = \tau/\mathtt{u}$, and detrended revenue is $Y_0^{-\alpha} z n^{\alpha}$.

(ii) The vacancy policy $v(z,n)$ satisfies the first order condition:
$$c_v(v(z,n);z,n) = q(\theta)\left[\phi S_n(z,n) + (1-\phi)\int_0^{S_n(z,n)} \left(S_n(z,n) - S_n'\right)\, dH_n(S_n')\right].$$

(iii) The law of motion for firm level employment is
$$\frac{dn}{dt}(z,n) = \begin{cases} -\frac{n}{dt} & n < n_E^*(z) \\ q(\theta)v(z,n)\left[\phi + (1-\phi)H_n(S_n(z,n))\right] - n\left[\delta + \lambda^E(\theta)(1 - H_v(S_n(z,n)))\right] & n \in \left[n_E^*(z), n_L^*(z)\right) \\ \frac{n_L^*(z) - n}{dt} & n \geq n_L^*(z), \end{cases}$$
where the notation $\frac{n}{dt}$ denotes a jump of size n, and where the exit threshold satisfies value-matching consistent with and the exit and layoff boundaries satisfy smooth-pasting conditions in productivity and employment:
$$\underbrace{S\big(z, n_E^*(z)\big) = \vartheta}_{\text{Value-matching from (8)}}\ ,\ \underbrace{S_z\big(z, n_E^*(z)\big) = 0\,,\ S_n\big(z, n_E^*(z)\big) = 0 \quad \text{if}\ \frac{dn}{dt}\big(z, n_E^*(z)\big) < 0\,,\ S_n\big(z, n_L^*(z)\big) = 0}_{\text{Smooth-pasting conditions from (8)}}$$

(iv) Vacancy- and employment-weighted distributions of marginal surplus are consistent:
$$H_v(S_n) = \int \mathbb{1}_{[S_n(z,n) \leq S_n]} \frac{v(z,n)}{\mathtt{v}} dH(z,n) \quad , \quad \mathtt{v} = \int v(z,n) dH(z,n)$$
$$H_n(S_n) = \int \mathbb{1}_{[S_n(z,n) \leq S_n]} \frac{n}{\mathtt{n}} dH(z,n) \quad , \quad \mathtt{n} = \int n\, dH(z,n)$$

(v) The measure of firms $H(z,n)$ is stationary, and admits a density function $h(z,n)$ that satisfies:

$$0 = -\frac{\partial}{\partial n}\left(\frac{dn}{dt}(z,n)\,h(z,n)\right) - \frac{\partial}{\partial z}\left(\mu(z)\,h(z,n)\right) + \frac{\partial^2}{\partial z^2}\left(\frac{\sigma(z)^2}{2}h(z,n)\right) + \mathtt{m}_0\,\pi_0(z)\,\mathbf{\Delta}(n)$$

where $\mathbf{\Delta}$ is the Dirac delta "function" which is zero everywhere except $n = n_0$ where it is infinite.

(vi) Entry $\mathtt{m}_0$ is such that the expected value of a new entrant is zero:

$$\eta c_0 = \int S(z,n_0)dF(z;\zeta),$$

where F is a Pareto distribution with tail parameter, and ζ is determined by imitation: $\zeta = \psi(2\mu/\sigma^2)$, with $\mu = -\alpha\left[(\overline{\mu}+g)+\sigma^2/2\right]z$

(vii) Vacancy meeting rate $q(\theta)$ and conditional probability of meeting an unemployed worker ϕ are consistent with the aggregate matching function given employment $\mathtt{n}$, unemployment ($\mathtt{u} = \bar{\mathtt{n}} - \mathtt{n}$), and vacancies $\mathtt{v}$.

(viii) The initial level of output in (8) is consistent with output of all firms in the economy under $H(z,n)$:

$$Y_0^\alpha = \mathtt{m}\int_{N\times Z} zn^\alpha dH(z,n)$$

and the rate of benefits $\widetilde{b}(\tau,\mathtt{u})$ is consistent with government budget balance $\widetilde{b}(\tau,\mathtt{u}) = \tau/\mathtt{u}$, and the interest rate is consistent with the household Euler equation $r = \rho + g$.

(ix) The market for the fixed factor clears. This requires $p_t^X = \eta Y_t = e^{gt}Y_0$. Under this condition the fixed factor is perfectly elastically supplied by households and hence the market for H_t clears at the quantity of the input demanded by firms.

(x) Prices p_{it} and quantities y_{it} are consistent with household demand (1) under $c_{it} = y_{it}$ for each good, which implies $C_t = Y_t$.

A.2 Methodology

Consider an alternative value ψ'. The equilibrium variables that enter the HJB for surplus (8) are: (i) growth g', via the drift in firm relative productivity $\mu' = -\alpha(\overline{\mu}+g')$, (ii) unemployment $\mathtt{u}'$, via the value of non-employment to workers $\widetilde{b}' = \tau/\mathtt{u}'$, as well as the share of potential hires who are unemployed, and (iii) the level of output Y_0', via the shifter in revenue. Given a guess of these three objects $(g',\mathtt{u}',Y_0')$ we can solve the detrended HJB equation (8) to obtain $S(z,n;g',\widetilde{b}',Y_0')$ and the associated stationary equilibrium of firms $H(z,n)$ following the approach in BEMV. We then check whether (i) the guess of unemployment is consistent with worker flows in the stationary equilibrium, (ii) the guess of Y_0' is consistent with the aggregation condition under the stationary distribution of firms $H(z,n)$: $Y_0'^\alpha = \mathtt{m}\int zn^\alpha dH(z,n)$, and (iii) the free entry condition holds under the implied distribution of entrant productivity which depends on ψ' and the distribution of incumbents via $\zeta' = \psi'(2\mu'/\sigma^2)$:

$$c_0 = \int S(z, n; g', \widetilde{b}', Y_0')dF(z, \zeta').$$

The key observation is that the free-entry condition under fixed `m` is used to solve for g' in the new BGP equilibrium. The following argument makes this clear. As we decrease ψ, entrants draw from a distribution that is skewed further toward the tail of incumbent productivities. This makes entry more valuable. As more entrants attempt to enter and entrants are better on average, costs of inputs are bid up, inducing more incumbent firms to exit. That is, the rate of obsolescence accelerates, increasing g. As relative productivity declines more quickly, however, the value of entry falls. The rate of growth g increases until the value of entry falls such that the free-entry condition is satisfied.